QUINTESSENCE MODEL AND COSMIC MICROWAVE BACKGROUND

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A particular kind of quintessence is considered, with equation of motion $p_Q/\rho_Q = -1$, corresponding to a cosmological term with time-dependence $\Lambda(t) = \Lambda(t_0)(R(t_0)/R(t))^P$ and we examine how values of Ω_m and Ω_Λ depend on P.

In this talk I summarize the paper[1]. We shall investigate the position of the first Doppler peak in the Cosmic Microwave Background (CMB) analysis using results published earlier[2].

The combination of the information about the first Doppler peak and the complementary analysis of the deceleration parameter derived from observations of the high-red-shift supernovae leads to fairly precise values for the cosmic parameters Ω_m and Ω_Λ. We shall therefore also investigate the effect of quintessence on the values of these parameters.

To introduce our quintessence model as a time-dependent cosmological term, we start from the Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \Lambda(t)g_{\mu\nu} + 8\pi G T_{\mu\nu} = 8\pi G \mathcal{T}_{\mu\nu} \tag{1}$$

where $\Lambda(t)$ depends on time as will be specified later and $T^\mu_\nu = \mathrm{diag}(\rho, -p, -p, -p)$. Using the Robertson-Walker metric, the '00' component of Eq.(1) is

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G\rho}{3} + \frac{1}{3}\Lambda \tag{2}$$

while the 'ii' component is

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -8\pi G p + \Lambda \tag{3}$$

Energy-momentum conservation follows from Eqs.(2,3) because of the Bianchi identity $D^\mu(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}) = D^\mu(\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}) = D^\mu \mathcal{T}_{\mu\nu} = 0$.

Note that the separation of $\mathcal{T}_{\mu\nu}$ into two terms, one involving $\Lambda(t)$, as in Eq(1), is not meaningful except in a phenomenological sense because of energy conservation.

In the present cosmic era, denoted by the subscript '0', Eqs.(2,3) become respectively:

$$\frac{8\pi G}{3}\rho_0 = H_0^2 + \frac{k}{R_0^2} - \frac{1}{3}\Lambda_0 \tag{4}$$

$$-8\pi G p_0 = -2q_0 H_0^2 + H_0^2 + \frac{k}{R_0^2} - \Lambda_0 \tag{5}$$

where we have used $q_0 = -\frac{\ddot{R}_0}{R_0 H_0^2}$ and $H_0 = \frac{\dot{R}_0}{R_0}$.

For the present era, $p_0 \ll \rho_0$ for cold matter and then Eq.(5) becomes:

$$q_0 = \frac{1}{2}\Omega_M - \Omega_\Lambda \tag{6}$$

where $\Omega_M = \frac{8\pi G\rho_0}{3H_0^2}$ and $\Omega_\Lambda = \frac{\Lambda_0}{3H_0^2}$.

Now we can introduce the form of $\Lambda(t)$ we shall assume by writing

$$\Lambda(t) = bR(t)^{-P} \tag{7}$$

where b **is** a constant and the exponent P we shall study for the range $0 \leq P < 3$. This motivates the introduction of the new variables

$$\tilde{\Omega}_M = \Omega_M - \frac{P}{3-P}\Omega_\Lambda, \quad \tilde{\Omega}_\Lambda = \frac{3}{3-P}\Omega_\Lambda \tag{8}$$

It is unnecessary to redefine Ω_C because $\tilde{\Omega}_M + \tilde{\Omega}_\Lambda = \Omega_M + \Omega_\Lambda$.

The equation for the first Doppler peak incorporating the possibility of non-zero P is found to be the following modification for $\Omega_C = 0$

$$l_1 = \pi\left(\frac{R_t}{R_0}\right)\left[\tilde{\Omega}_M\left(\frac{R_0}{R_t}\right)^3 + \tilde{\Omega}_\Lambda\left(\frac{R_0}{R_t}\right)^P\right]^{1/2}\int_1^{\frac{R_0}{R_t}}\frac{dw}{\sqrt{\tilde{\Omega}_M w^3 + \tilde{\Omega}_\Lambda w^P}} \tag{9}$$

The dependence of l_1 on P is illustrated graphically by figures in [1]. Further discussion of the dependence of l_1 on Ω_C, using the formulas of [2], is given in [3].

We have introduced P as a parameter which is real and with $0 \leq P < 3$. For $P \to 0$ we regain the standard cosmological model. But now we must investigate other restrictions already necessary for P before precision cosmological measurements restrict its range even further.

Only for certain P is it possible to extrapolate the cosmology consistently for all $0 < w = (R_0/R) < \infty$. For example, in the flat case $\Omega_C = 0$ which our universe seems to approximate, the formula for the expansion rate is

$$\frac{1}{H_0^2}\left(\frac{\dot{R}}{R}\right)^2 = \tilde{\Omega}_M w^3 + \tilde{\Omega}_\Lambda w^P \tag{10}$$

This is consistent as a cosmology only if the right-hand side has no zero for a real positive $w = \hat{w}$. The root $\hat{w}$ is

$$\hat{w} = \left(\frac{3(1-\Omega_M)}{P - 3\Omega_M}\right)^{\frac{1}{3-P}} \tag{11}$$

If $0 < \Omega_M < 1$, consistency requires that $P < 3\Omega_M$.

Another constraint on the cosmological model is provided by nucleosynthesis which requires that the rate of expansion for very large w does not differ too much from that of the standard model.

The expansion rate for $P = 0$ coincides for large w with that of the standard model so it is sufficient to study the ratio:

$$\begin{aligned}(\dot{R}/R)^2_P/(\dot{R}/R)^2_{P=0} \quad &\stackrel{w\to\infty}{\to} \quad (3\Omega_M - P)/((3-P)\Omega_M) &(12)\\ &\stackrel{w\to\infty}{\to} \quad (4\Omega_R - P)/((4-P)\Omega_R) &(13)\end{aligned}$$

where the first limit is for matter-domination and the second is for radiation-domination (the subscript R refers to radiation).

The constraints of avoiding a bounce ($\dot{R} = 0$) in the past, and then requiring consistency with BBN leads to $0 < P < 0.2$.

Clearly, from the point of view of inflationary cosmology, the precise vanishing of $\Omega_C = 0$ is a crucial test and its confirmation will be facilitated by comparison models such as the present one.

1. J.L. Crooks, J.O. Dunn, P.H. Frampton and Y.J. Ng. `astro-ph/0005406`
2. P.H. Frampton, Y.J. Ng and R.M Rohm, Mod. Phys. Lett **A13,** 2541 (1998). `astro-ph/9806118`.
3. S. Weinberg, `astro-ph/0006276`.